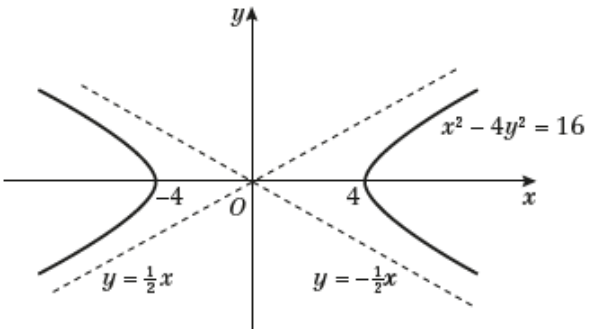
Exercise 2B

1 a $\frac{x^2}{16}-\frac{y^2}{4}=1$

$a=4, b=2$

Asymptotes $y=\pm\frac{1}{2}x$

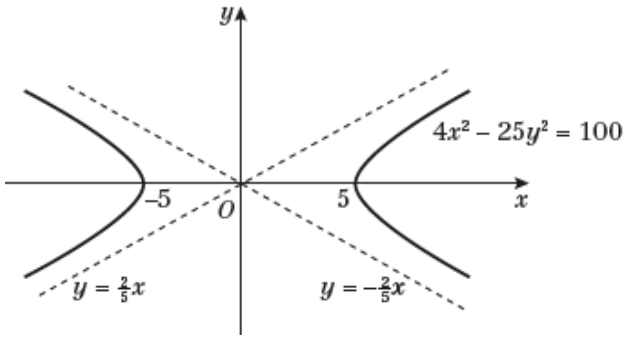


b $4x^2-25y^2=100$

$\Rightarrow \frac{x^2}{25}-\frac{y^2}{4}=1$

$a=5, b=2$

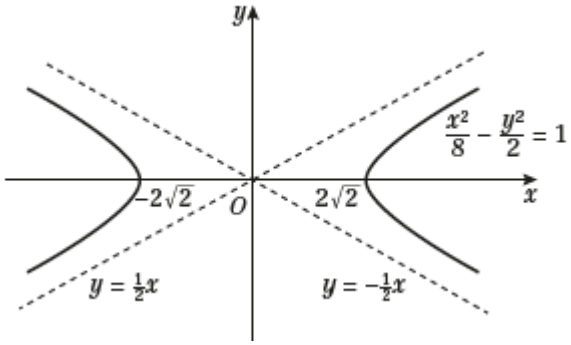
Asymptotes $y=\pm\frac{2}{5}x$



c $\frac{x^2}{8}-\frac{y^2}{2}=1$

$a=2\sqrt{2}, b=\sqrt{2}$

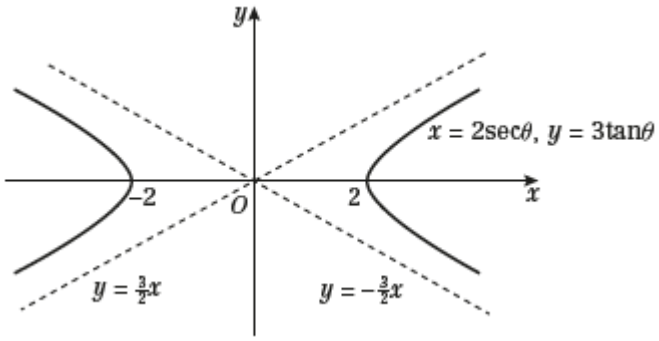
Asymptotes $y=\pm\frac{1}{2}x$



2 i a $x=2\sec\theta, y=3\tan\theta$

$a=2, b=3$

Asymptotes $y=\pm\frac{3}{2}x$

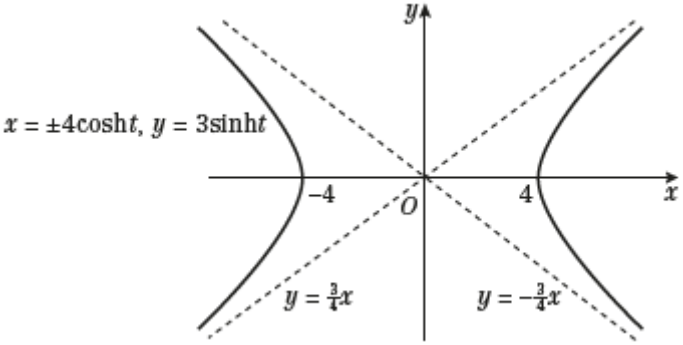


b $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{4}-\frac{y^2}{9}=1$

ii a $x=\pm4\cosh t, y=3\sinh t$

$a=4, b=3$

Asymptotes $y=\pm\frac{3}{4}x$

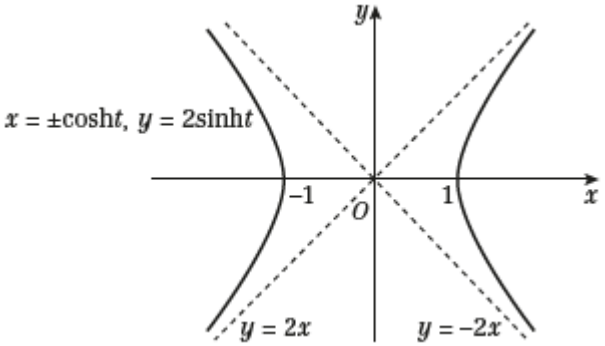


b Equation: $\frac{x^2}{16}-\frac{y^2}{9}=1$

2 iii a $x = \pm\cosh t, y = 2\sinh t$

$a = 1, b = 2$

Asymptotes $y = \pm 2x$

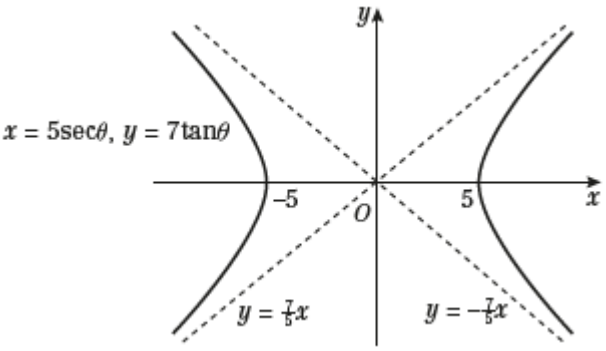


b Equation: $x^2 - \frac{y^2}{4} = 1$

iv a $x = 5\sec\theta, y = 7\tan\theta$

$a = 5, b = 7$

Asymptotes $y = \pm\frac{7}{5}x$



b Equation: $\frac{x^2}{25} - \frac{y^2}{49} = 1$

Challenge

A general point of the hyperbola is $\begin{pmatrix} ct \\ \frac{c}{t} \end{pmatrix}$

We apply the rotation matrix $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ and we get $\begin{pmatrix} \frac{ct}{\sqrt{2}} - \frac{ct}{\sqrt{2}} \\ \frac{ct}{\sqrt{2}} + \frac{ct}{\sqrt{2}} \end{pmatrix}$,

so $x^2 = \frac{ct^2}{2} - c^2 + \frac{ct^2}{2}$ and $y^2 = \frac{ct^2}{2} + c^2 + \frac{ct^2}{2}$

Then we can compute

$$y^2 - x^2 = \frac{c^2t^2}{2} + c^2 + \frac{c^2t^2}{2} - \left(\frac{c^2t^2}{2} - c^2 + \frac{c^2t^2}{2}\right)$$
$$= 2c^2$$

Then the rotated hyperbola satisfies an equation $y^2 - x^2 = a^2$, with $a = \pm c\sqrt{2}$